CALCULATING CREDIT RISK FOR A PORTFOLIO OF FIXED-RATE BONDS

Aretina Magdalena David-Pearson,

University"Politehnica", Bucharest, *e-mail:* aretina_david@yahoo.co.uk

Abstract. *Credit risk is defined and explained, as well as the two models of credit risk measurement. The used 60 credit risk factors that are used are presented, together with the credit worthiness index model according to which they are used. After data description and analysis, the models used in the scenarios generation are described in detail. A portfolio of fixed rate bonds has been taken and its credit risk has been calculated.*

Keywords: *calculating credit risk, fixed rate bonds portfolio.*

1.INTRODUCTION

Credit risk is defined, in its strong version, as the risk that an obligor will not be able to meet its financial obligations towards its creditors. Under this definition default is the only credit event. The weaker definition of credit risk is based on market perception. This definition implies that obligors will face credit risk even if they do not fail their financial obligations yet but the market perceives they might fail in the future. This is known as the *mark-to-market* definition of credit risk and gives rise to migration as well as default as possible credit events. Perception of financial distress gives rise to credit downgrade.

There are two categories of credit risk measurement models: Counterparty Credit Exposure models and Portfolio Credit Risk models. Counterparty exposure is the economic loss that will be incurred on all outstanding transactions if a counterparty defaults, unadjusted by possible future recoveries. Counterparty exposure models measure and aggregate the exposures of all transactions with a given counterparty. Although simple to implement, the model has been widely criticized because it does not accurately account for stochastic exposures. Since exposures of derivatives such as swaps depend on the level of the market when default occurs, models must capture not only the actual exposure to counterparty at the time of the analysis but also its potential future changes. By simulating counterparty portfolios through time over a wide range of scenarios, these models explicitly capture the contingency of the market on derivative portfolios and credit risk.

In contrast, Portfolio Credit Risk (*PCR*) models measure credit capital and are specifically designed to capture portfolio effects, *specifically obligor correlations*. It accounts for the benefits of diversification. With diversification, the risk of the portfolio is different from the sum of risk across counterparties. Correlations allow a financial institution to diversify their portfolios and manage credit risk in an optimal way. However, empirical work shows generally that all *PCR* models yield similar results if the input data is consistent. A major limitation of all current *PCR* models is the assumption that market risk factors, such as interest rates, are deterministic. Hence, they do not account for stochastic exposures. One of the objectives of *PCR* is to measure *economic* capital.

Credit risk is the risk of losses caused by counterparty or an issuer defaulting on their payment obligations. It focuses on the calculations of credit exposures and capital allocation based on estimated economic losses. A portfolio of Fixed Rate bonds of Depfa Bank Plc (www.depfa.com) has been taken. The total exposure of all the instruments for this portfolio is approximately 635 billion USD.

2.THE CREDIT WORTHINESS INDEX MODEL

Credit Worthiness Index (*CWI*) comprises systemic credit risk arising from movements in the credit drivers that are common to the counterparties in the portfolio, and idiosyncratic (un-systemic) risk that is specific to a particular counterparty in the portfolio.

The correlation infrastructure and data consist of credit drivers that we have identified to be relevant for measuring credit risk for any credit portfolio with global coverage, the idiosyncratic risk, and the sensitivities of representative counterparties for any given global region and industry sector.

For a portfolio of J obligors, the credit quality of an obligor is modelled through a multi-factor Credit Worthiness Index Y_j and is described in following equation

$$Y_j(t) = \sum_{i=1}^{I} \beta_{ij} Z_i(t) + \alpha_j \varepsilon_j \qquad (1)$$

In the above equation, the systemic credit risk component of the *CWI* for each obligor j, j=1,2,...J , is assumed to be driven by credit drivers Z_i , i=1,2..., I . Each credit driver Z_i represents the country and industry sector affiliation of obligor j . The sensitivities vector can then be written as $(\beta_{1j}, \beta_{2j}, ..., \beta_{ij}, ..., \beta_{Ij})$ where β_{ij} is the sensitivity of obligor j to credit driver i. The second term in equation above represents the obligor-specific, idiosyncratic risk component. The higher the sensitivities of an obligor to a credit driver or a set of them, the higher its systemic risk and the lower its idiosyncratic risk will be.

$$\alpha_j = \sqrt{1 - \sum_{i=1}^{I} \beta_{ij}^2} = \sqrt{1 - R^2_j} \qquad (2)$$

R_j^2 is the proportion of variance of Y_j explained by the credit drivers, and ε_j in equation (1) are independent standard normal variables. Also, in equation (1) the index Y_j is standard normal in view of equation (2); it has zero mean and unit variance. One refers to α_j as the specific weight for counterparty j.

In the above framework, correlation between any two obligors' credit quality is governed by the correlations among the risk drivers in their *CWI*s. This is represented by the joint Variance – Covariance matrix in this paper.

The covariance between the *CWI*s Y_l and Y_k of any two obligors can be written as:

$$\operatorname{cov}(Y_l, Y_j) = \sum_{i=1}^{I} \beta_{il}\beta_{kj} Cov(Z_i, Z_k) + \sum \beta_{il}\beta_{ij} Var(Z_i) \quad (3)$$

Under a multi-factor *CWI* model, the sensitivity β_{ij} must be estimated for a set of credit drivers appropriate for a given counterparty.

We have also assumed that when all except one of the sensitivities to the credit drivers equal zero, the sensitivities vector for obligor *j* is with systemic impact of only one credit driver, namely, Z_i. In this case, although the systemic risk in each obligor *CWI* is determined by one single index, the model still captures the diversification effect across different region-industry sector pairs through the different indexes serving as proxies for such pairs. At the same time, correlated defaults are captured to the extent that the indexes themselves are correlated.

Once we estimate the *CWI* for each counter party in the credit portfolio, the occurrence of default or migration in each one of a set of Monte Carlo scenarios on the credit drivers can be simulated. Also, given other information such as counterparty exposures, recovery rates, length of planning horizon, etc., the portfolio loss distribution is computed where the losses now incorporate the impact of correlated defaults and migrations. A pre-specified loss percentile, e.g., 99.90th, of the loss distribution then signifies the credit risk economic capital for the portfolio.

In theory, the above process requires prior knowledge of the specific counterparty names in the credit portfolio in order to compute the credit loss distribution. But in the above infrastructure and framework, however, one eliminates this requirement of prior knowledge of specific names. Instead, one estimate CWIs for *representative counterparties* on to which specific counterparties in any given portfolio may be mapped, based on a set of well-defined criteria.

3.DATA DESCRIPTION AND ANALYSIS

A portfolio of Fixed Rate bonds comprising of approximately 1,700 counterparties and 140 portfolios denominated in different currencies with different credit ratings assigned to each one of them has been taken, with USD as the base currency.

A sample of market and credit risk factors has been considered. Some risk factors may influence both market and credit risk. Interest rates, for example, are market prices determining the values of various fixed income instruments, but they also have an influence on default probabilities, and they are in turn influenced by idiosyncratic properties of individual obligors.

Scenarios have been generated on market risk factors like Treasury interest rates and Credit risk factors like the systemic credit drivers. In this portfolio 50,000 scenarios have been generated on each market and credit state drivers for 2 time steps of 89 days and 365 days.

The credit risk has been measured with respect to credit drivers’ indices which are the systemic macroeconomic factors that have an impact on the credit risk of the bond instruments for each counter party.

One has to make a distinction between regulatory and economic capital since regulatory capital does not take into account correlations among obligors that form a portfolio and so, depending on the credit quality of the obligors to which a financial institution is exposed to, it might over- or under-estimate economic capital requirements.

As we have seen above, we assume a multi-factor *CWI* model wherein a given counterparty has a single credit driver associated to it. Given our objective of arriving at an infrastructure whereby credit losses can be computed for an arbitrary credit portfolio that may contain counterparties from any part of the globe, we have first identified a comprehensive set of credit drivers with global coverage. In addition to providing comprehensive coverage, the selected credit drivers also capture the characteristics of the economic and credit environment of a given region as well as the characteristics of the particular industry within which a given counterparty is operating.

Accordingly, we have divided the globe into three regions to start with: Americas, Asia-Pacific, and Europe. We have taken a dominant economy in each region like Japan in Asia-Pacific, and U.K. in Europe but for Americas we have taken a combination of other ten globally diversified Indices. We have divided the globe in to five regions.

Further, within each region, we have utilized the GICS system for 10 industry sectors in order to account for their distinct credit risk characteristics. Thus, we have a total of 50 region-sector combinations to cover a global credit portfolio or a portfolio covering one or more of the five regions.

Regions	**Sectors**
1. Americas	1. Energy
2. UK	2. Basic Materials
3. Europe (ex UK)	3. Industrials
4.Japan	4. Consumer Cyclical
5. Asia-Pacific (ex Japan)	5. Consumer Non-Cyclical
	6. HealthCare
	7. Financials
	8. Information Technology
	9.Telecommunication Services
	10. Utilities

We have also taken the following 10 Indexes to represent additional Credit Drivers to capture the macroeconomic impact of the overseas countries

1. FTSE/ASE 20 (FTASE) Index – The FTASE Index consists of 20 of the largest and most liquid stocks that trade on the Athens Stock Exchange. It was developed in September 1997 out of a partnership between the Athens Stock Exchange and FTSE International.
2. BRAZIL BOVESPA STOCK Index (IBOV) - The Bovespa Index is a total return index weighted by traded volume and is comprised of the most liquid stocks traded on the Sao Paulo Stock Exchange.
3. CAC Index – The CAC-40 Index is a narrow-based, modified capitalization-weighted index of 40 companies listed on the Paris Bourse.

4. DAX Index – The German Stock Index is a total return index of 30 selected German blue chip stocks traded on the Frankfurt Stock Exchange.
5. KOSPI Index – The KOSPI Index is a capitalization-weighted index of all common shares on the Korean Stock Exchanges.
6. MEXICO BOLSA Index (MEXBOL) – The Mexican Bolsa Index is a capitalization-weighted index of the leading stocks traded on the Mexican Stock Exchange.
7. NIKKIE 225 (NKY) Index – The Nikkei-225 Stock Average is a price-weighted average of 225 top-rated Japanese companies listed in the First Section of the Tokyo Stock Exchange.
8. S&P/TSX COMPOSITE Index (SPTSX) – The S&P/Toronto Stock Exchange Composite Index is a capitalization-weighted index designed to measure market activity of stocks listed on the TSX.
9. S&P 500 Index (SPX) – Standard and Poor's 500 Index is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all major industries.
10. MSCI TAIWAN Index (TWY) – The MSCI Taiwan Index is a market capitalization-weighted index of stocks listed on the Taiwan Stock Exchange.

For each region-sector combination, we designate the corresponding Dow Jones Region-Sector Index as the credit driver for counterparty affiliated to that region and sector. In order to estimate the sensitivity of a representative counterparty for a given region-sector combination to the Region-Sector Index as the credit driver, we have carried out the following steps. First, there have been collected the time series data on all fixed rate corporate bonds in a given region and sector and the time series data on the corresponding Region-Sector indexes.

Next, one uses the average of the estimated R-Squares for the bonds in the given region-sector combination obtained when the individual fixed income returns are regressed on the Index returns, which are then available for use as the R-Square for a representative counterparty in a given region-sector combination.

Once R-Square estimates are available for a representative counterparty for every region-sector combination, equation (2) above can be used to estimate the sensitivity and the specific risk for any counterparty in a given region and sector therein.

4.INTEREST RATE RISK FACTORS MODELLED WITH BLACK-KARASINSKI MODEL

The Black-Karasinski simulation model is a normal, mean-reverting model that is applied on the log of the risk factors. This model is used for simulations over a large time horizon that uses arbitrary length time steps. I have applied the Black-Karasinski Model for Interest Rate Treasury risk factor block.

At each step, a mean-reverting process takes a small step toward a target mean before taking a random step up or down. The long-term range or bounds on the values of the mean-reverting process depend on the values of both the rate of mean reversion and the volatility.

The reasons why the Black Karasinski mean reversion simulation model has been used for the interest rate risk factors include the following:

- Each country has an ideal or equilibrium interest rate term structure
- When the term structure is out of equilibrium, a variety of market, economic, and political forces act to bring interest rates back to this equilibrium
- These forces become stronger as the term structure deviates further from equilibrium

Theoretical Model:

$$Y(t) = \log X(t)$$

$$dY(t) = a(\overline{Y} - Y(t))\,dt + \sigma\, dW(t)$$

Simulation Equation:

$$Y(t+\Delta t) = \bar{Y} + (Y(t) - \bar{Y})e^{-a\Delta t} + \sigma\sqrt{\frac{(1-e^{-2a\Delta t})}{2a}}\eta$$

Calibration Equation

$$Y(t_t) = \log(X(t_t))$$

$$V(t_t) = Y(t_t) - Y(t_t - 1) - (1 - e^{-a/N})(\bar{Y} - Y(t_t - 1))$$

In the Interest rate block one has the time series data for Zero Coupon Treasury Interest Rate curves for monthly data from 31/01/1997 until 31/01/2007 for major currencies of AUD, CHF, DKK, EUR, GBP, and USD. We have the data for the above time series in the form of treasury interest rates for one week, month, quarter, half year, three quarter, one year, two years, four years, five years, seven years, ten years, fifteen years, twenty years, twenty five years and thirty years. We can observe the mean reverting level under the lognormal model and the rate of mean reversion for all the interest rate risk factors in the figure given below. The average mean reverting level is 4.3337% and average rate of mean reversion is 8.7883

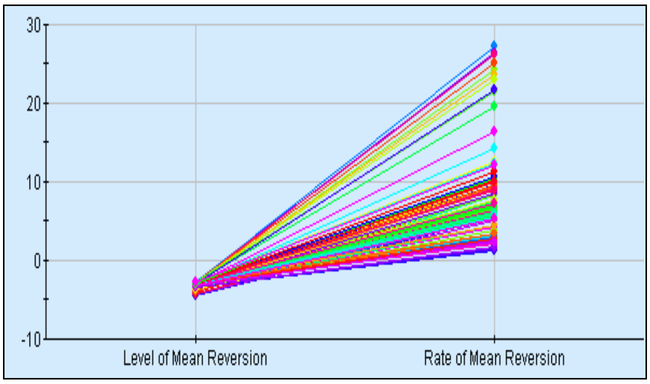

Fig.1. Mean Reverting Level of Interest Rate Risk Factors

5.CREDIT RISK FACTORS MODELLED WITH GEOMETRIC BROWNIAN MOTION MODEL

The Geometric Brownian Motion Model, also known as the Black-Scholes Model with zero drift, is used to generate multi-step Monte Carlo scenarios. It is a stochastic process that is similar to the Brownian Motion Model, but is pictured as a geometric random walk in continuous time. The distribution of these continuously compounded returns at the end of any finite time interval will be a *LogNormal distribution*. It also includes an adjustment based on Ito's Lemma. This process generates scenarios whose dispersion increases without bound as simulated time elapses. Stochastic Process:

$$dX(t) = \sigma X(t) dW(t)$$

Simulation Equation:

$$X(t) = X(t-\Delta t).\exp\left\{-\frac{1}{2}\sigma^2 \Delta t + \eta\sqrt{\Delta t}\right\}$$

Calibration Equation:
The increments have a normal distribution.

$$V(t_t) = \ln\left(\frac{X(t_t)}{X(t_{t-1})}\right) + \frac{1}{2}\sigma^2 \Delta t$$

The Geometric Brownian Motion Model has been applied for Credit Drivers risk factor block. In this block we have the time series data for independent credit drivers in the form of market indexes for monthly data from 31/05/2000 until 31/05/2007 for major indexes regions across the globe etc and for different sectors within these regions like Energy, Basic Materials, Industrials, Consumer Cyclical, Consumer Non-Cyclical, Health Care, Financials, Information Technology, Telecommunication Services and Utilities. In total we have 60 credit drivers comprising of combination of different worldwide indexes with one of these sectors above.

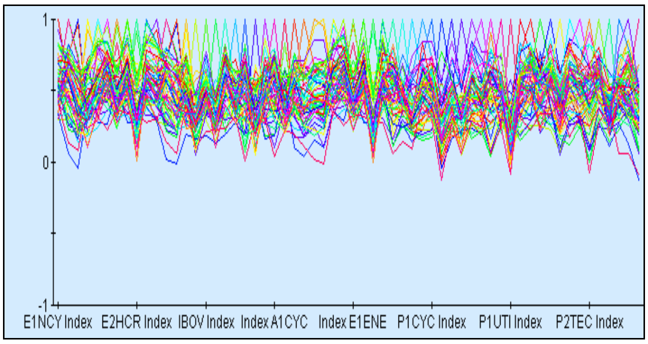

Fig. 2. Correlation movements of the credit drivers

The correlation behaviour between the 60 credit risk factors observed in observed in *Figure 2* has all the 60 credit risk factors on the *x*-axis and their correlation with the other indexes on the *y*-axis. The correlation ranges between 0 and 1.

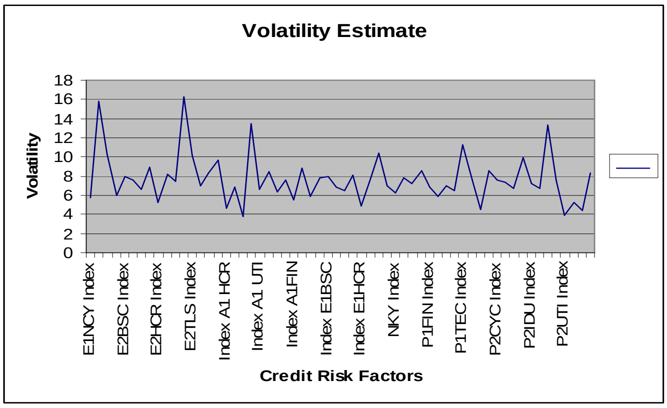

Fig.3. Credit risk factors' volatility

6.RESULTS

We have calculated the measures for Expected Shortfall and Credit VaR at 99.95 percentile level of confidence for both time steps 89 days and 365 days. The result of a Value at Risk (*VaR*) calculation for a portfolio indicates the amount of money that can be lost over a given time horizon at a specific confidence interval.

The results for the Credit risk estimates can be observed in the below table:

Credit Risk	89 days	365 days
ExpectedShortfall@99.95	183,943,506.80	237,456,252.20
Credit VaR @99.95	137,898,249.00	198,598,398.50
Mean Loss	1,101,751.04	8,101,601.48
MeanExposure	1,889,172,428.00	1,470,622,964.00

The other results estimates for Credit Risk are

- Mean Exposure – This is the average exposure per counterparty for the given portfolio of fixed rate bonds
- Mean Loss – This is the average expected loss and can be arrived as per the following calculations: (Mean Exposure * Expected Probability of Default * Assumed deterministic rate or recovery)
- Expected Shortfall (ES) - Expected Shortfall is the average of all losses that meet or exceed the VaR. For this reason, it is also known as "conditional VaR" or "tail conditional expectation."

The expected shortfall for Credit risk has been calculated, which is 184 million for 89 days and 237 million for 365 days.

We can observe that the credit VaR at 99.95 percentile for 89 days is approximately 138 million, which is 0.02% of the overall portfolio exposure and 199 million for 365 days which is 0.03% of the overall portfolio exposure.

On one side, this might be due to the portfolio consisting of counterparties with very less stochastic probability to default in spite of having all sorts of credit ratings. On the other side, the inputs for the credit sensitivity used for this analysis are coming from an analysis in progress.

7. REFERENCES

[1] Aziz, J. and N. Charupat, 1998, "Calculating credit exposure and credit loss: a case study," Algo Research Quarterly, 1(1): pp.31-46.

[2]Jarrow, R.A. and Turnbull S.M. (1995). Pricing Derivatives on Financial Securities Subject to Credit Risk. Journal of Finance, Vol. 50, No. 1, pp. 53-85.

[3]Kijima, M. and Muromachi Y. (2000). Evaluation of Credit Risk of a Portfolio with Stochastic Interest Rate and Default Processes. Journal of Risk, Vol. 3,pp. 5-36